Chewing Bubblegum and Enumerating Partitions

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Integer Partitions

How many ways can we add positive integers to make 10?

In other words, how many **partitions** of 10 are there?

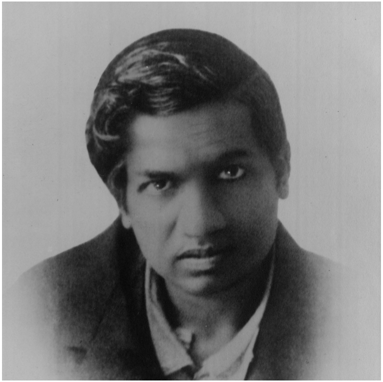
Who Has Studied Partitions?

Leonhard Euler

Who Has Studied Partitions?

G. H. Hardy

Who Has Studied Partitions?



Srinivasa Rammanujan

Who Has Studied Partitions?

Freeman Dyson

Who Has Studied Partitions?

Tamsyn Morrill

Why Study Partitions?

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And **this** is a *q*-hypergeometric series: $\sum_{n=0}^{\infty} \frac{(a;q)_n (b;q)_n}{(c;q)_n (q;q)_n} z^n$.

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The coefficients of a q-hypergeometric series count partitions. So if you can count, you can prove results in q-hypergeometric series!

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The answer is 42.

Thank you!